Flexible beam analysis of the effects of string tension and frame stiffness on racket performance

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Abstract

It is generally accepted that a decrease in string tension leads to greater racket power and an increase in tension improves racket control. The increase in power at low string tension can be attributed partly to a decrease in energy loss in the ball and partly to a decrease in the vibrational energy transferred to the racket. Racket control is affected if the ball strikes the strings towards one edge of the frame, in which case the racket will rotate about the long axis through the handle. The angle of rotation is decreased when the string tension is increased. Quantitative estimates of the magnitude of these effects are presented, using a one dimensional flexible beam model to describe the racket and springs to model the ball and strings. For tensions in the range 50–60 lb (220–270 N), commonly used in tennis rackets, and for a ball incident at right angles to the string plane, changes in racket power and control are essentially negligible. However, a significant increase in racket power can be achieved by increasing the stiffness of the racket frame.

Keywords: tennis, string tension, power, control

Nomenclature

e	Coefficient of restitution
e_A	Apparent coefficient of restitution
E	Young's modulus
I	Area moment of inertia
I_x	Moment of inertia about the x-axis
k_1	Spring constant of ball in compression
k_2	Spring constant of ball in expansion
k_s	Spring constant of strings
m	Mass of beam segment
m_b	Mass of ball
s	length of beam segment
v_1	Incident speed of ball
v_2	Rebound speed of ball
x	Coordinate along the long (x) axis

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y_b	Displacement of centre of mass of ball
y_n	Displacement of beam at impact point
Y_b	Compression of ball
Y_s	Displacement of strings
Y_T	$Y_s + Y_b$
θ	Rotation angle of racket about x-axis
τ	Impact duration

Introduction

In this paper, a theoretical model is presented to describe the effects of string tension and frame stiffness on racket power and control. It is well known that the effect of decreasing the string tension in a tennis, squash or badminton racket is to generate more power and that high string tension leads to better ball control. However, the theoretical basis to support these observations is not well developed and quantitative estimates are not available to make a comparison with experimental results (Elliot 1982; Groppel *et al.* 1987; Knudson 1991a, b, 1997; Thornhill *et al.* 1993). The relationship between racket power and string tension can be explained in part in terms of energy losses in the ball. When a ball impacts on the strings, some of the impact energy is momentarily stored as elastic energy in the ball and the strings. If a racket is strung at a lower tension, then more of the impact energy is stored in the strings and less is stored in the ball. Since the ball dissipates a much larger fraction of its elastic energy than the strings, the result is an increase in racket power, as described by Brody (1979). The energy coupled to vibrations of the frame is also reduced when the string tension is reduced.

The term 'control' is generally understood to refer to the ability of a player to hit the ball along a desired trajectory (Brody 1987, 1997). String tension may affect control in several different ways. If the ball is incident at right angles to the string plane, then the player might expect the ball to rebound at right angles. However, if the ball impacts towards one edge of the frame, then the racket will rotate about its long axis during the impact, and the ball will not rebound along the expected path. The angle of rotation can be reduced by increasing the string tension, since the rotation angle is proportional to the impact duration. Even if the ball impacts at normal incidence in the centre of the strings, the ball will not rebound back along the incident path unless the ball is struck at exactly the right moment. Given that the impact duration is typically about 5 ms, and the angular velocity of a racket may be as high as 30 rad s^{-1}, the racket could swing through an angle of about 9° during this time. The angle can be decreased by increasing the string tension, the rotation angle in this case also being proportional to the impact duration.

The collision between a ball and a racket is often modelled as a rigid body collision, using an assumed or measured coefficient of restitution to account for the energy losses. In this paper, the racket is modelled as a one-dimensional flexible beam in order to determine separately the energy losses in the ball and the racket, and to determine how these quantities vary with string tension and frame stiffness. The model was recently used to analyse the impact of a ball with aluminium beams of various lengths and stiffness (Cross 1999a). Excellent agreement was obtained with the experimental data, which indicated that the collision dynamics are strongly affected by wave propagation along the beam. Despite the fact that rackets are hand-held under normal playing conditions, it is assumed in this paper that the racket is freely suspended. Previous experimental and theoretical results show that this is a good approximation since the ball usually leaves the strings before the impulse is reflected off the hand and back to the impact point (Brody 1997; Cross 1999a).

Theoretical model

The interaction of a ball with the strings of a racket is illustrated in Fig. 1. It is assumed throughout this paper that the ball is incident at right angles to the string plane. The strings are stretched by the impact force and exert a restoring force on the ball, and an equal and opposite force on the racket head. The ball and the strings can both be modelled as springs, as shown in Fig. 2. It is assumed that the strings are massless since the string mass is much less than the mass of the racket frame. The ball is modelled as a mass, m_b, attached to a nonlinear spring, the spring constants being different during the compression and expansion phases in order to account for hysteresis losses in the ball.

The force exerted by the ball on the strings is transmitted to the racket frame around the perimeter of the head by all the strings. Despite the fact that the racket head is round and the frame is hollow, the vibration modes, and the node locations of such a racket can be adequately modelled by assuming that the racket behaves as a rectangular cross-section beam (as depicted in Fig. 2). The zero frequency dynamics of the racket (i.e. its rotation, translation and the location of the centre of percussion) can also be determined in terms of a rectangular beam model since the conservation equations for momentum and energy are independent of the shape of the racket. The resultant force on the racket can be assumed to act at a single point on the beam, with a line of action along the line of incidence of the ball, as illustrated in Fig. 3. This is consistent with the fact that frame vibrations are not excited by an impact at the vibration node near the middle of the strings. A more complete description of the impact dynamics would require a two or even three dimensional model of the racket, for example to account for the two dimensional structure of the node line in the string plane (Kawazoe 1997).

The equation of motion of the ball is given by

$$d^2y_b/dt^2 = -F/m_b \quad (1)$$

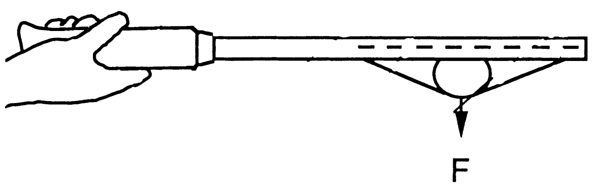

Figure 1 Impact of a ball on the strings of a racket.

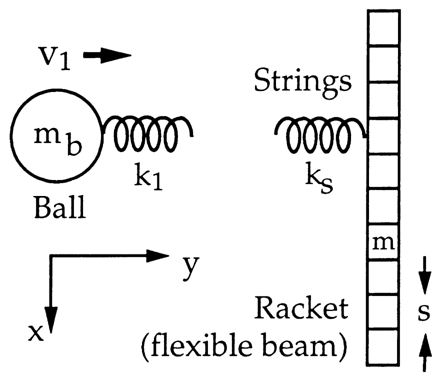

Figure 2 Model of the impact assuming that the ball and strings behave as springs and the racket behaves as a flexible beam. For numerical purposes, the racket is divided into a large number of small segments, each of mass m. The spring constant of the ball is k_1 while it compresses and is given by Eq. (4) when it expands.

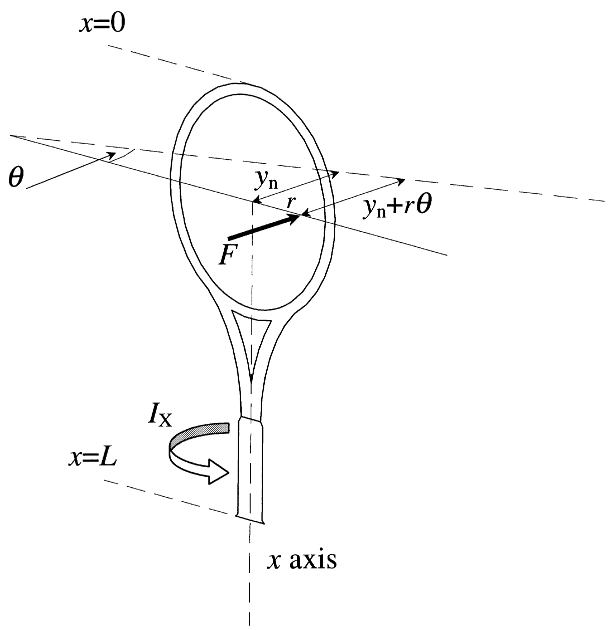

Figure 3 A force F acting at a distance r from the long axis causes the racket to rotate through an angle θ about the long axis.

where y_b is the displacement of the centre of mass of the ball, and F is the force exerted by the strings on the ball. If the string plane is displaced by a distance Y_s relative to the frame during the impact then F is given by

$$F = k_s Y_s \tag{2}$$

where k_s is the spring constant of the strings. The elastic properties of the ball can be modelled by assuming that

$$F = k_1 Y_b \tag{3}$$

during the compression phase and

$$F = k_2 Y_b^p \tag{4}$$

during the expansion phase, where Y_b is the compression of the ball, F is the force acting on both the ball and the strings and p is a parameter describing the effect of hysteresis in the ball. If k_1 and k_2 are constants and if Y_o is the maximum compression of the ball during any given impact, then $k_1 Y_o = k_2 Y_o^p$ so $k_2 = k_1 Y_o^{p-1}$. The hysteresis loss in the ball is equal to the area enclosed by the F vs. Y_b curve for a complete compression and expansion cycle. The actual force laws for a tennis ball are more complicated than the simplified expressions here (Cross 1999b), but the parameter p can be chosen to give a total ball loss equal to an experimentally determined loss, as described below. The net effect of the impact on the ball is determined primarily by the total impulse acting on the ball, the precise details of the force waveform being relatively unimportant. Similarly, the relative amplitudes of the various vibration modes excited in the racket depend more on the impact duration rather than the the precise details of the force waveform. This is clearly the case for a tennis racket since the impact duration is too long to excite any modes other than the fundamental mode.

The equation of motion for a beam subject to an external force, F_o per unit length, has the form (Goldsmith 1960; Graff 1975)

$$\rho A \frac{\partial^2 y}{\partial t^2} = F_o - \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) \tag{5}$$

where ρ is the density of the beam, A is its cross-sectional area, E is Young's modulus, I is the area moment of inertia, and y is the transverse displacement of the beam at coordinate x along the beam. For a uniform beam of mass M and length L, numerical solutions of Eq. (5) can be obtained by dividing the beam into N equal segments each of mass $m = M/N$ and separated in the x direction by a distance $s = L/N$. An impacting ball may exert a force acting over several adjacent segments, depending on the ball diameter and the assumed number of segments. For simplicity it was assumed that the ball impacts on only one of the segments, exerting a time-dependent force, F, as given by Eqs (2)–(4). The equation of motion for that segment (the nth segment) is obtained by multiplying all terms in Eq. (5) by s, in which case

$$m \frac{\partial^2 y_n}{\partial t^2} = F - (EIs) \frac{\partial^4 y_n}{\partial x^4} \tag{6}$$

assuming that the beam is uniform so that E and I are independent of x. The equation of motion for the other segments is given by Eq. (6) with $F = 0$. A similar procedure could be used to model a head heavy or a head light racket, but it is assumed in this paper that the racket is neutrally balanced and has a uniform mass distribution. The boundary conditions at a freely supported end are given by $\partial^2 y/\partial x^2 = 0$ and $\partial^3 y/\partial x^3 = 0$. The coordinate $x = 0$ is taken at the tip of the racket and the coordinate $x = L$ locates the end of the handle.

Angular rotation of the beam about its long (x) axis, illustrated in Fig. 3, is described by the relation

$$\mathrm{d}^2\theta/\mathrm{d}t^2 = Fr/I_x \tag{7}$$

where θ is the angle of rotation, r is the distance from the impact point to the x-axis and I_x is the moment of inertia of the racket for rotation about the x-axis. For simplicity, it is assumed that the beam undergoes rigid body rotation about the x-axis but remains flexible for transverse displacements along the x-axis. This assumption can be justified in part by the fact that fundamental frequency for torsional oscillations of a racket is much higher than the fundamental frequency for transverse oscillations (Kawazoe 1997). Even though the racket is hand held in practice, the opposing torque exerted by the hand will be typically much smaller than that exerted by the impacting ball. The peak force exerted by the ball can be larger than 1000 N.

Equation (6) describes the transverse y displacement of the long axis as a function of time and as a function of the coordinate x along the axis. The transverse y displacement of the impact point is given by $y_n + r\theta$. For the model shown in Figs 2 and 3, the compression of the ball plus the compression of the strings is therefore given by

$$Y_T = y_b - (y_n + r\theta) \tag{8}$$

The compression of the strings is given by Eq. (2) and the compression of the ball is given by Eq. (3) or (4). By equating Eqs (2) and (3), or (2) and (4), it is easy to show that

$$Y_b = k_s Y_T / (k_1 + k_s) \tag{9}$$

during the compression phase, and

$$Y_b + k_2 Y_b^p / k_s = Y_T \tag{10}$$

during the expansion phase.

It is assumed that at $t = 0$, $y_b = 0$, $y = 0$ for all beam segments, the beam is initially at rest and that $dy_b/dt = v_1$. The subsequent motion of the ball and the beam was evaluated numerically using finite difference forms of Eqs (1), (6) and (7). These results were used to determine the rebound speed of the ball, v_2, and the apparent coefficient of restitution (ACOR), $e_A = v_2/v_1$. In normal play, the racket is swung towards the ball and is not normally at rest at the moment of impact. The resulting outgoing speed of the ball is easily related to laboratory measurements or to theoretical estimates of the ACOR for an initially stationary racket, using a simple coordinate transformation as described below.

Ball parameters

From Eqs (3) and (4), the energy loss in the ball, E_b, is given by

$$E_b = \oint F dY_b = \frac{(p-1)k_1 Y_o^2}{2(p+1)} \tag{11}$$

This relation can be used to determine an appropriate value of p from the known coefficient of restitution, e, for the ball. For example, if a ball of mass m_b is incident normally on a concrete slab at speed v_1 and it rebounds at speed v_2 then $e = v_2/v_1$ and the energy loss in the ball is $E_b = 0.5\ m_b(v_1^2 - v_2^2)$. For an impact on concrete, the energy stored in the ball at maximum compression is given by $0.5\ k_1 Y_o^2 = 0.5\ m_b v_1^2$ and hence

$$1 - e^2 = \frac{E_b}{0.5\, m_b v_1^2} = \frac{(p-1)}{(p+1)} \tag{12}$$

The rules of tennis specify that a tennis ball must have a mass of 56.7 ± 0.9 g, and that $e = 0.745 \pm 0.017$ for a drop height of 100 inches (254 cm) onto a concrete slab, in which case p lies in the range $2.44 < p < 2.77$. For the calculations presented below, it is assumed that $k_1 = 3 \times 10^4\ \mathrm{N\ m^{-1}}$ and $p = 2.55$. For these parameters the impact duration for a 100-inch drop onto concrete is 4.64 ms, and $e = 0.751$. For a drop on the strings of a head-clamped racket with $k_s = 2 \times 10^4\ \mathrm{N\ m^{-1}}$, the solutions of Eqs (1)–(4) indicate that the impact duration is extended to 6.98 ms, and $e = 0.908$. If k_s is increased to $4 \times 10^4\ \mathrm{N\ m^{-1}}$, the impact duration is 5.90 ms and $e = 0.866$. The increase in both e and the impact duration, compared with a drop on concrete, is consistent with observations (Brody 1979). The coefficient of restitution is known to vary slightly with ball speed, in which case p could be taken as a function of ball speed. However, the main objective in this paper is to determine the effects of varying k_s, taking $p = 2.55$ as a typical value.

String parameters

Energy losses in the strings are ignored in this paper since they are negligible compared with losses in the ball (Brody 1995). An additional simplification is made by assuming that the spring constant of the strings, k_s, is independent of the string plane deflection. The deflection of the string plane has been measured by several authors (Brody

1979, 1987; Leigh & Lu 1992), at least for an applied force up to about 250 N. For a small applied force, the deflection is linearly proportional to the applied force and k_s is typically between 2×10^4 and 6×10^4 N m^{-1} for string tensions in the range 178 N (40 lb) to 356 N (80 lb). Under these conditions, k_s is independent of the string type or the string diameter and is directly proportional, in any given racket, to the string tension, T.

For a fast serve or smash, the peak force on the strings is considerably larger than 250 N. The peak force can be estimated by assuming that a ball of mass 57 g is accelerated from 0 to 60 m s^{-1} in 4 ms. The time-average force is then 855 N, and the peak force is about double this, or 1710 N. Under these conditions, the strings behave as slightly nonlinear springs, since the strings stretch and the tension increases during the impact. The increase in k_s is typically about 10% for large deflections of the string plane (Cross 2000a). However, all calculations presented below are given for conditions where k_s is assumed to remain constant in time. Results are presented for $k_s = 2 \times 10^4$ N m^{-1} and $k_s = 4 \times 10^4$ N m^{-1}, corresponding to a factor of two change in the initial string tension for small deflections of the string plane and approximately a factor of two change in the initial string tension for large deflections of the string plane. Since a factor of two change in k_s was found to have only a small effect on the ball speed, any effect due to the nonlinear nature of the strings is clearly of little consequence.

More generally, k_s also varies with position over the string plane. The string plane stiffness is relatively constant over most of the string plane but it increases substantially near the racket frame. Allowing k_s to remain constant over the string plane does not limit the analysis to the central region of the string plane, since additional solutions are also given below for conditions where k_s is varied over several orders of magnitude.

Racket parameters

For the purpose of this study, we will consider a graphite/epoxy composite tennis racket of length $L = 71$ cm and mass $M = 340$ g. The fundamental vibration frequency of such a racket, when freely suspended, is typically about 125 Hz (period $T = 8$ ms). If the racket is treated as a uniform beam, then EI is equal to 150 Nm for these parameters, regardless of the exact beam cross-section. The frame in most modern rackets is in fact constructed from hollow sections, but the relevant factor in Eq. (6) is EI, not the cross-sectional dimensions. The vibration frequency of a racket decreases slightly as the string tension is increased, but this effect is neglected in the calculations presented below. The moment of inertia about the long axis was taken as $I_x = 0.0015$ kg m^2, which is typical for such a racket (Brody 1985).

Energy balance

Numerical solutions of Eqs (1)–(10) are shown in Fig. 4 for an impact at $x = 10$ cm, $r = 5$ cm on an initially stationary racket with $k_s = 4 \times 10^4$ N m^{-1}. The beam was numerically divided into $N = 39$ segments. The numerical accuracy of the solutions

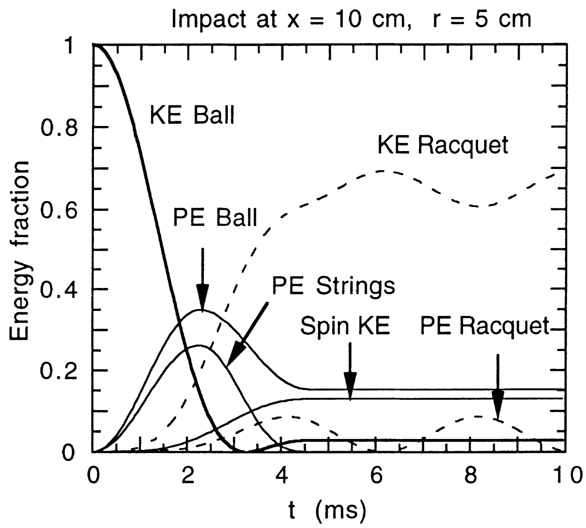

Figure 4 Distribution of the initial ball energy as a function of time when $k_s = 4 \times 10^4$ N m^{-1} for an impact at $x = 10$ cm from the tip of the racket and $r = 5$ cm from the long axis. Each curve is normalised to the initial ball energy. The impact duration $\tau = 4.78$ ms and $e_A = 0.166$.

was checked by ensuring that the total energy during and after the impact remained constant. The various energy components are shown in Fig. 4, all components being normalised to the initial kinetic energy of the incident ball, as follows:

(a) The 'KE racket' curve represents the total instantaneous kinetic energy of all beam segments. The 'KE racket' curve therefore includes the kinetic energy due to rotation of all segments about a transverse axis through the centre of mass, as well as the kinetic energy due to translation of all segments, plus the kinetic energy due to vibration of the racket.
(b) The 'PE racket' curve represents the total instantaneous potential energy of all beam segments, resulting from beam bending. The energy stored in the racket as a result of vibrations is given by the peak value of the 'PE racket' curve after the impact is over.
(c) The 'Spin KE' curve represents the energy of rotation of the racket about the long axis resulting from the off-axis impact.
(d) The 'PE Ball' curve is the energy stored in the ball as a result of its compression plus the energy dissipated in the ball during its expansion phase.
(e) As far as the player is concerned the only useful component is the 'KE Ball' fraction which is 1.0 at $t = 0$, and 0.028 after the collision, corresponding to an ACOR $e_A = 0.166$.

As shown in Fig. 4, most of the initial kinetic energy of the ball is converted to potential energy of the ball and strings during the compression phase of the impact. As the ball and strings expand, and during several vibration cycles after the ball leaves the strings, the energy is redistributed in a manner that is consistent with conservation of energy and momentum for the system. The results show that the only vibration mode of any significance is the fundamental mode at 125 Hz, which is excited when the ball impacts at any location other than the vibration node near the centre of the strings. It might appear in Fig. 4 that the frequency is 250 Hz, but the vibration energy is a maximum twice each cycle.

Results for impacts along the x-axis

Figure 5 shows the variation of e_A, as well as the fractional energy loss in the ball and the fractional energy loss due to vibration of the racket, as a

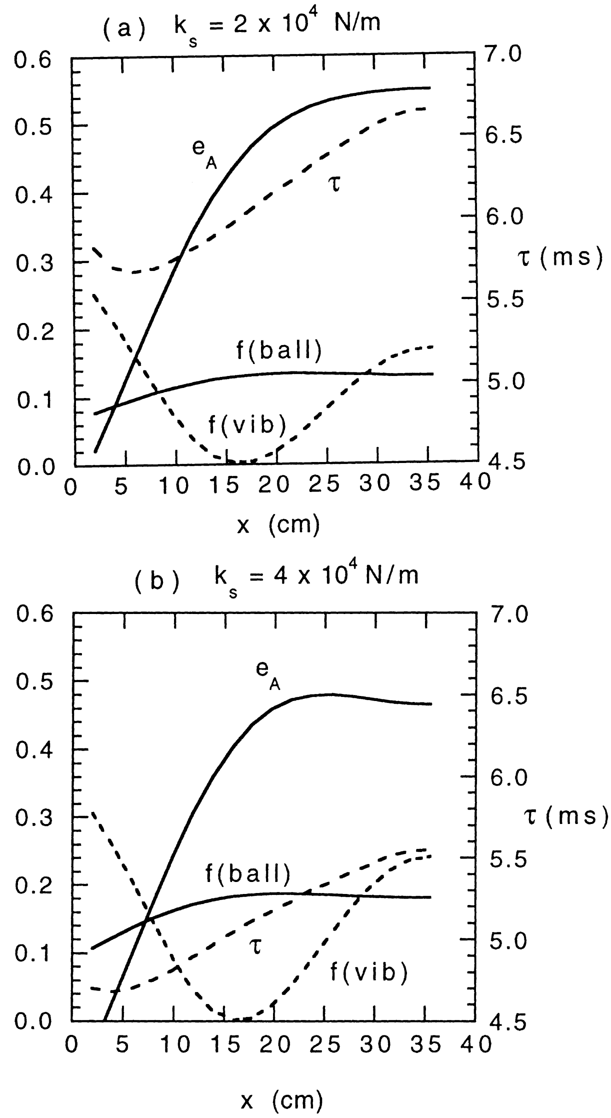

Figure 5 Impact parameters plotted as a function of impact distance, x, from the tip of the racket when $r = 0$ and (a) $k_s = 2 \times 10^4$ N m^{-1} and (b) $k_s = 4 \times 10^4$ N m^{-1}. f(ball) denotes the fraction of the initial ball energy dissipated in the ball, and f(vib) denotes the fraction stored as vibrational energy in the racket. τ is the impact duration (right scale) and e_A is the ratio of the rebound speed to the incident speed of the ball, assuming the racket is initially at rest.

function of the impact point along the x-axis when $r = 0$. Results are shown for $k_s = 2 \times 10^4$ N m^{-1} and $k_s = 4 \times 10^4$ N m^{-1}. Both graphs show the same general features, where e_A is zero at a 'dead' spot near the tip of the racket and increases to a broad maximum near the throat of the racket. This behaviour is easily demonstrated experimentally, at least in a qualitative sense, simply by dropping a ball at various spots on the strings and observing the bounce. The graphs also show that the energy loss due to vibration is essentially zero for an impact at the fundamental vibration node near the middle of the strings, at $x = 16$ cm. The vibration loss and the ball loss are similar in magnitude at impact points removed from the node, and are both reduced for impacts on the softer strings. As a result, e_A is larger at all impact points for soft strings than for stiff strings. The impact duration, τ, is also increased for an impact on the softer strings. The impact duration is smaller near the tip of the racket than near the throat since the tip accelerates away from the ball more rapidly than the throat.

The change in e_A with k_s is relatively small considering that there is a factor of two difference in k_s in Fig. 5(a),(b). For example, for an impact at the centre of the strings, $e_A = 0.44$ for the softer strings and $e_A = 0.41$ for the stiffer strings. This corresponds to a 7% change in the ball speed when it bounces off a racket that is initially at rest or moving slowly compared with the speed of the incident ball. Since the ball rebounds at speed $v_2 = e_A v_1$, then in a reference frame where the ball is initially at rest, the racket will be incident at speed v_1 and the ball will come off the strings at speed $v = v_1 + v_2 = (1 + e_A)v_1$. In this case, corresponding to a serve or overhead smash, the above increase in e_A results in only a 2% increase in the ball speed. A decrease in string tension from 60 lb (270 N) to 50 lb (220 N), would therefore result in an increase in serve speed of only 0.7%. Similarly, in a reference frame where the racket speed is equal to the incident ball speed, the ball will come off the strings at speed $v = (1 + 2e_A)v_1$. This is typical of a forehand or backhand, in which case a factor of two decrease in k_s leads to a 3.3% increase in the rebound speed of the ball. A decrease in string tension from 60 to 50 lb would therefore lead to an increase in ball speed of only 1.1%.

Figure 6 shows the same parameters as in Fig. 5, plotted as a function of string plane stiffness for an impact 6 cm from the tip of the racket at $r = 0$. The stiffness is varied over three orders of magnitude in this diagram, in order to produce significant changes in the parameters. An interesting result is that the vibration amplitude can be reduced essentially to zero if the stiffness is reduced sufficiently so that the impact duration exceeds about 10 ms. The amplitudes of the various vibration modes of a beam depend not only on the impact location but also on the impact duration. A short impact of duration τ contains a continuous spectrum of frequency components up to a frequency $f \sim 1/\tau$. The amplitude of the spectrum peaks at zero frequency and drops to zero near $f = 1/\tau$. For example, if the impact duration is 5 ms, the frequency spectrum extends to about 200 Hz, which is above the 125 Hz vibration frequency of the fundamental mode but well below the second mode at 345 Hz. Consequently, the second mode is not usually excited in a tennis racket. If the impact duration is increased by reducing the string

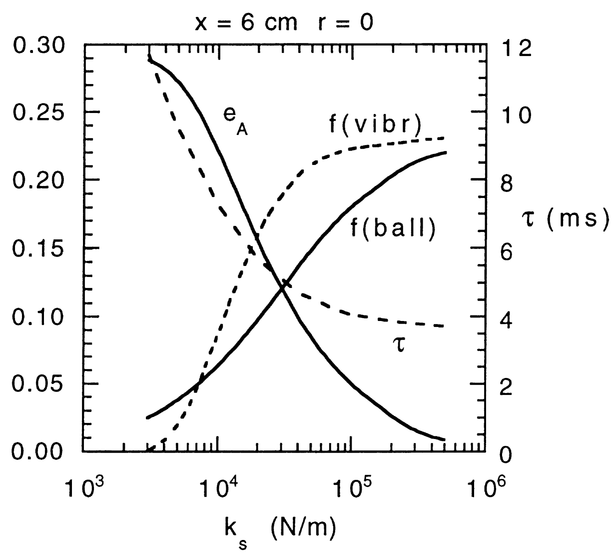

Figure 6 The same parameters in Fig. 5 plotted as a function of k_s for an impact near the tip of the racket at $x = 6$ cm, $r = 0$.

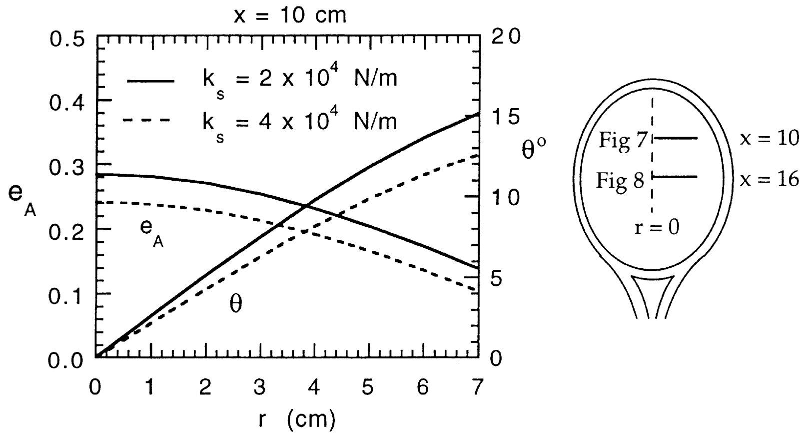

Figure 7 Effect on e_A and θ of an off-axis impact at $x = 10$ cm, where θ is the angle of rotation about the long axis at the end of the impact.

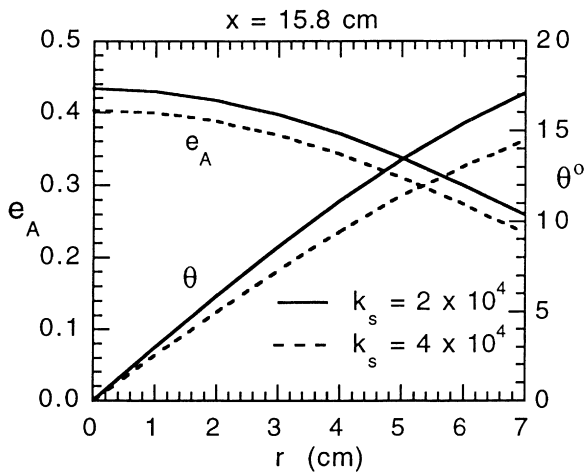

Figure 8 Effect on e_A and θ of an off-axis impact at $x = 15.8$ cm.

tension, then the spectrum extends to a lower limit, and the amplitude of the induced 125 Hz vibration is reduced as shown in Fig. 6.

Results for off-axis impacts

If the ball impacts towards one edge of the racket frame, then the racket will rotate about the x-axis as shown in Fig. 3. The rotation angle increases during the impact by an amount that is directly proportional to the incident ball speed, v_1, and it is approximately proportional to the impact distance, r, from the x-axis. The rotation angle, θ, at the end of the impact period is plotted as a function of r in Figs 7 and 8 for a case where $v_1 = 30$ m s^{-1} and where $k_s = 2 \times 10^4$ N m^{-1} or $k_s = 4 \times 10^4$ N m^{-1}. In Fig. 7, the ball impacts at a distance $x = 10$ cm from the tip of the racket, while in Fig. 8 the ball impacts at $x = 15.8$ cm. The variation of e_A with r is also shown in Figs 7 and 8.

Regardless of the impact location, e_A increases when k_s is reduced, and the rotation angle about the x-axis increases. The changes in e_A and angle with string tension are relatively small considering the factor of two difference in k_s in Figs 7 and 8. For example, the rotation angle increases by about 19%, at any given impact point, when k_s is decreased by a factor of two. This percentage increase matches the percentage increase in impact duration (from about 5.0 ms to 6.0 ms at the centre of the strings). Consequently, an increase in string tension from 50 to 60 lb would reduce the rotation angle for an off-axis impact by about 4% and it would also decrease the impact duration by about 4%.

A change from 50 to 60 lb tension should therefore be barely noticeable, at least in terms of the ball rebound speed and angle. A ball impacting at $r = 4$ cm near the centre of the strings causes the racket to rotate by about 10° if $v_1 = 30$ m s^{-1}. This is likely to generate a significant error in ball placement. However, the error will not be significantly different if the racket rotates by 9.6° rather than 10°, which would be the effect of changing the string tension from 50 to 60 lb. A much more significant reduction in the error could be achieved by weighting the frame in order to increase I_x, since the rotation angle is inversely proportional to I_x.

Effects of varying the frame stiffness

The amplitude of the fundamental vibration mode of the racket frame can be reduced almost to zero, while maintaining an impact duration of about 5 ms, if the frame stiffness is increased so that the fundamental vibration frequency is near or above 200 Hz. Many 'wide body' rackets have a frequency of about this value. Calculations for a such a racket are shown in Fig. 9, assuming that the fundamental vibration frequency is 200 Hz ($EI = 384$ Nm) and that the mass and length of the racket are the same as that in Fig. 6. Compared with the vibration loss shown in Fig. 6, the vibration loss for a wide body racket is much smaller for impacts near the tip or throat of the racket and e_A is therefore significantly larger. For example, at $k_s = 4 \times 10^4$, e_A is increased from 0.10 to 0.18, which translates (because of the $1 + e_A$ factor), to a 7% increase in serve speed from a point near the tip of the racket.

Many elite players serve from a point near the tip of the racket, presumably because of the added height advantage and because the racket is moving fastest at the tip. The ability to serve at high speed from a point near the tip is possibly the main reason that modern rackets appear to be more powerful than the old wooden rackets of 20 years ago. There is no significant difference in racket power between stiff and flexible rackets for an impact at the vibration node since the vibration amplitude is essentially zero in both cases. Given the significant power advantages at other impact locations, and the reduction in frame and handle vibrations, it is surprising that wide body rackets are not as popular as they were several years ago. This can partly be explained by the fact that the stiffness of narrow rackets has been increased in recent years using more advanced graphite composite materials. It is conjectured that professional players do not require the extra power of wide body rackets, or that they have not learned to control it, and that the average player thinks that narrow rackets are better because the professionals use them.

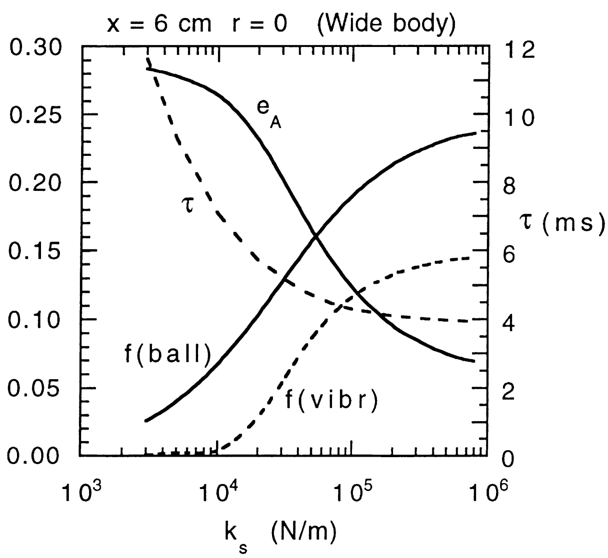

Figure 9 As for Fig. 6 but for a stiffer racket with a fundamental vibration frequency of 200 Hz. The racket in Figs 4–8 vibrates at 125 Hz.

Discussion

The effects of varying the string tension on racket power and control, as described above, are broadly consistent with experimental results, given the relatively large uncertainties and some inconsistencies associated with published data (Elliot 1982; Groppel *et al.* 1987; Knudson 1991a, b, 1997; Thornhill *et al.* 1993). However, they are much smaller than one might have guessed by reading the popular literature on the subject. Elite players often report that a small change in string tension, either in the initial tension or in the subsequent drop in tension after several matches, has a large effect on the performance or 'playability' of a racket. They also describe old strings as being dead or lacking the power and control of new strings. This is not consistent with the above calculations, nor is it consistent with the fact that the energy loss in both new and old strings is essentially negligible (Cross 2000a). One might suspect the calculations, but the result for a case where the ball impacts at the vibration node is easily verified. Vibration losses in the racket can then be neglected, in which case simple mass and spring models can be used to describe both the ball and the racket (Cross 2000b).

Players sometimes comment that ball control is improved if the ball remains in contact with the strings for a longer period. This is also inconsistent with the above calculations. Since a factor of two decrease in k_s increases the impact duration by about 1 ms, and since k_s is approximately proportional to the string tension, a decrease in string tension from 60 to 50 lb will increase the impact duration by about 0.2 ms. If the racket is rotating at a relatively high angular velocity, say 30 rad s^{-1}, this change in tension changes the angle of rotation during the impact by about 0.34°. This is barely significant, given that (a) the player could change the time of the initial impact by 0.2 ms to achieve the same result and (b) the racket head more commonly moves in an approximate straight line path during the impact, rather than in a circular path, in which case the angular speed at impact is much less than 30 rad s^{-1}. If the racket head moves in a straight line path, any change in impact duration has no effect at all on the initial rebound path of the ball.

The analysis described in this paper has been restricted to impacts at normal incidence. At other angles of incidence, the dynamics will depend on the coefficient of friction between the ball and the strings. The coefficient of friction will affect the change in ball speed in a direction parallel to the string plane and it will affect the ball spin and rebound angle. The effects of string tension on power and control, reported by players, could possibly be explained if the coefficient of friction depends on the string tension. Alternatively, other string parameters such as the elasticity or the amount of wear and tear may affect both the string tension and the coefficient of friction. These effects warrant further investigation.

Conclusions

Effects of varying the string tension and frame stiffness in a tennis racket have been considered in this paper using a flexible beam model to describe the racket, and springs to model the ball and strings. A significant increase in racket power can be achieved by increasing the stiffness of the racket frame, at least for impacts near the tip or throat of the racket. There is no significant increase in power for an impact at the vibration node near the centre of the strings since the fundamental vibration mode is not excited. Racket power can also be increased by stringing the racket at a lower tension, but the effect is almost negligible. For impacts near the centre of the strings it was found that a factor of two decrease in string plane stiffness yields a 7% increase in the apparent coefficient of restitution. This translates to a 2% increase in serve speed. Consequently, if the string tension in a tennis racket is decreased from 60 to 50 lb, the increase in serve speed will be only about 0.7%. In the case of a forehand or backhand, the corresponding increase in ball speed is about 1.1%. Off-axis impacts result in rotation of the racket about the long axis. The rotation angle is typically about 10° for a ball impacting 4 cm off-axis and incident at 30 m s^{-1} relative to the racket. The rotation angle can be reduced by about 20% by increasing the string plane stiffness by a factor of two, or by about 4% if the tension is increased from 50 to 60 lb.

References

Brody, H. (1979) Physics of the tennis racket. *American Journal of Physics*, **47**, 482–487.

Brody, H. (1985) The moment of inertia of a tennis racket. *Physics Teacher*, **23**, 213–216.

Brody, H. (1987). *Tennis Science for Tennis Players*. University of Pennsylvania Press, Philadelphia, PA, USA.

Brody, H. (1995) How would a physicist design a tennis racket? *Physics Today*, **48**, 26–31.

Brody, H. (1997) The physics of tennis. III. The ball–racket interaction. *American Journal of Physics*, **65**, 981–987.

Cross, R. (1999a) Impact of a ball with a bat or racket. *American Journal of Physics*, **67**, 692–702.

Cross, R. (1999b) Dynamic properties of tennis balls. *Sports Engineering*, **2**, 23–33.

Cross, R. (2000a) Physical properties of tennis strings. *3rd ISEA International Conference on the Engineering of Sport, Sydney.*, in press.

Cross, R. (2000b) Dynamic properties of tennis strings. *1st International Congress on Tennis Science and Technology, London*, 2000, in press.

Elliot, B.C. (1982) The influence of racket flexibility and string tension on rebound velocity following dynamic impact. *Research Quarterly for Exercise and Sport*, **53**, 277–281.

Goldsmith, W. (1960) *Impact*. E. Arnold, London, UK, pp. 108–129.

Graff, K.F. (1975) *Wave Motion in Elastic Solids*. Oxford University Press, Oxford, UK, pp. 140–210.

Groppel, J.L., Shin, I.-S., Spotts, J. & Hill, B. (1987) Effects of different string tension patterns and racket motion on tennis racket-ball impact. *International Journal of Sport Biomechanics*, **3**, 142–158.

Kawazoe, Y. (1997) Experimental identification of a hand-held tennis racket and prediction of rebound ball velocity in an impact. *Theoretical and Applied Mechanics*, **46**, 177–188.

Knudson, D. (1991a) Effect of string type and tension on ball vertical angle of rebound in static tennis impacts. *Journal of Human Movement Studies*, **20**, 39–47.

Knudson, D. (1991b) Effect of string tension and impact location on ball rebound accuracy in static tennis impacts. *Journal of Applied Biomechanics*, **9**, 143–148.

Knudson, D. (1997) The effect of string tension on rebound accuracy in tennis impacts. *International Sports Journal*, **1**, 108–112.

Leigh, D.C. & Lu, W.-Y. (1992) Dynamics of the interactions between ball, strings and racket in tennis. *International Journal of Sports Biomechanics*, **8**, 181–206.

Thornhill, P.M., Baker, J.S. & Cooper, S.M. (1993) Interaction between tennis string type and tension and the subsequent effect upon ball rebound velocity. *Journal of Human Movement Studies*, **24**, 157–167.